# A Comparison of the Brinson and Parilux Attribution Analysis Methods

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The Brinson method is a well-known method to decompose the excess return of a portfolio, relative to its benchmark portfolio, into components *security selection* and *sector selection*[1]. Security selection is the excess return resulting from the portfolio manager's selection of particular securities within sectors. Sector selection is the excess return resulting from the overweighting or underweighting of sectors relative to the benchmark portfolio. Parilux uses a method that is similar to the Brinson method, but which we believe has important advantages. The purpose of this paper is to explain the differences between these two methods, and show why we believe the Parilux method is preferable.

## A Review of the Brinson Method

The original paper describing the Brinson method is

> Brinson, Gary P., and Nimrod Fachler, "Measuring Non-US Equity Portfolio Performance," *Journal of Portfolio Management*, Spring 1985, pp. 73-76.

Online sources include:

> http://en.wikipedia.org/wiki/Performance_Attribution
>
> http://www.mscibarra.com/research/articles/2002/PerfBrinson.pdf

We can derive the Brinson model by starting with the equation for portfolio excess return ($ER$):

$$ER = r_p - r_b \tag{1}$$

where $r_p$ is the return of the portfolio during the analysis period, and $r_b$ is the return of the benchmark. We can compute these returns from the component sector returns and weights (beginning of period) as follows:

$$ER = \sum_i w_{pi} r_{pi} - \sum_i w_{bi} r_{bi} \tag{2}$$

where $r_{pi}$ is the return of portfolio sector $i$, $w_{pi}$ is the weight of portfolio sector $i$, $r_{bi}$ is the return of benchmark sector $i$, and $w_{bi}$ is the weight of benchmark sector $i$. Rearranging the first we can get[2]

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[1] These names can vary among different authors or practitioners, especially depending on the classification system that is being used. For example, when analyzing equity portfolios "security selection" might be called "stock selection". When analyzing multi-asset class portfolios "sector selection" might be called "asset allocation", "allocation", or "allocation effect", and "security selection" might be called "selection" or "selection effect". Here we mean for "sectors" to refer to any useful classification system such as asset classes, equity sectors, etc., and securities to refer to individual investments within sectors.

[2] Note that the Brinson method is sometimes expressed using the following intermediate quantities: $Q1 = \sum w_{bi} r_{bi} = r_b$, $Q2 = \sum w_{pi} r_{bi}$, $Q3 = \sum w_{bi} r_{pi}$, $Q4 = \sum w_{pi} r_{pi} = r_p$. Then sector selection is $Q2 - Q1$, security selection is $Q3 - Q1$, and "interaction" is $Q4 - Q3 - Q2 + Q1$.

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\begin{aligned}
ER = &\left[\left(\sum_i w_{pi} r_{bi}\right) - r_b\right] && \text{(Sector selection)} \\
&+ \left[\left(\sum_i w_{bi} r_{pi}\right) - r_b\right] && \text{(Security selection)} \\
&+ \left[r_p - \left(\sum_i w_{pi} r_{bi}\right) - \left(\sum_i w_{bi} r_{pi}\right) + r_b\right] && \text{(Interaction)}
\end{aligned} \qquad (3)
$$

The first bracketed quantity, defined as sector selection, is the excess return that would have been achieved based on the portfolio sector weights had the portfolio sector returns matched the benchmark sector returns. The second bracket quantity, defined as security selection, is the excess return that would have been achieved based on the portfolio sector returns had the portfolio sector weights matched the benchmark sector weights. The last bracketed quantity, defined as "interaction", "captures the value added that is not attributable solely to the asset allocation and stock selection decisions" (quoted from Wikipedia). We find this definition somewhat unsatisfying.

Taking an example from the second online link above (by Damien Laker) we have the following inputs:

| Sector | Benchmark | | Portfolio | |
|---|---|---|---|---|
| | Weight | Return | Weight | Return |
| 1 | 20% | 2.00% | 10% | 2.00% |
| 2 | 30% | 3.00% | 30% | 4.00% |
| 3 | 50% | 4.00% | 60% | 9.00% |
| Total | 100% | 3.30% | 100% | 6.80% |

The attribution results can then be calculated as from equation (3) as:

| | |
|---|---|
| Sector Selection | 0.20% |
| Security Selection | 2.80% |
| Interaction | 0.50% |
| Total Excess Return | 3.50% |

We can see that a majority of excess return is due to security selection, mainly from the strong performance in sector 3.

The Brinson method can also be applied to individual sectors in order to obtain results for each sector that total to the total portfolio results presented above. Breaking up $r_b$ and $r_p$ in equation 4 by sector and rearranging a bit we can obtain:

$$ER = \left[\sum_i (w_{pi} - w_{bi}) r_{bi}\right] \quad \text{(Sector Selection)}^3$$
$$+ \left[\sum_i w_{bi}(r_{pi} - r_{bi})\right] \quad \text{(Security selection)} \qquad (4)$$
$$+ \left[\sum_i (w_{pi} - w_{bi})(r_{pi} - r_{bi})\right] \quad \text{(Interaction)}$$

Now it is easier to see how to calculate the contributions for the individual sectors. We can see that a sector will contribute positive sector selection when a sector with a positive benchmark return is over-weighted, or when a sector with negative benchmark return is underweighted. Similarly we can see that a sector will contribute positive security selection whenever the portfolio outperforms the benchmark in the sector (assuming a positive benchmark weight). Expanding the above example to include individual sector contributions we have:

| | Sector 1 | Sector 2 | Sector 3 | Portfolio |
|---|---|---|---|---|
| Sector Selection | -0.20% | 0.00% | 0.40% | 0.20% |
| Security Selection | 0.00% | 0.30% | 2.50% | 2.80% |
| Interaction | 0.00% | 0.00% | 0.50% | 0.50% |
| Total Excess Return | -0.20% | 0.30% | 3.40% | 3.50% |

## Problems with the Brinson Method

One problem we see with the Brinson method is that the Interaction term is just not very illuminating. It seems like a fudge factor to account for the difference between the attributed returns and the actual total excess return. There has been quite a bit of discussion over the meaning and interpretation of this term.[4] We have recently become aware of a published paper by Stephen Campisi[5] that strongly agrees with our views of this term, as well as other problems with the Brinson method. In his conclusion Campisi states:

> *Despite the appearance of mathematical rigor in its calculation, investors and clients understand that interaction is not a legitimate part of the investment process. We believe this is because the interaction effect has always been an unspecified residual resulting from an incorrect analysis of the [security] selection effect. By correcting the measurement of the selection effect, the*

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[3] Some implementations of the Brinson method instead define sector selection as $\left[\sum_i (w_{pi} - w_{bi})(r_{bi} - r_b)\right]$, which we regard as a significant improvement.

[4] For example, see Laker, Damien, "What is this Thing Called Interaction?" *Journal of Performance Measurement*, Fall 2000, pp. 43-57.

[5] Stephen Campisi, CFA, "Debunking the Interaction Myth" *Journal of Performance Measurement*, Summer 2004, pp. 63-70.

*unnecessary complication of an interaction effect has been removed; leaving an analysis that is simpler, yet more accurate and representative of the active management process. This eliminates an aspect of attribution that has always been difficult to understand and to communicate to clients.*

A potentially more serious problem is that the Brinson method can result in questionable assessments of security and sector selection. To see the problem with sector selection consider the case where the portfolio contains only a tiny allocation to sector 1, say 0.01%, which could represent some residual holding of a liquidating investment. With such a low weight the portfolio sector 1 return is immaterial to the total portfolio return or total excess return. However, it is not immaterial to the Brinson security selection calculation. Consider this modified example:

| Sector | Benchmark | | Portfolio | |
|---|---|---|---|---|
| | Weight | Return | Weight | Return |
| 1 | 20% | 2.00% | 0.01% | 2.00% |
| 2 | 30% | 3.00% | 30.00% | 4.00% |
| 3 | 50% | 4.00% | 69.99% | 9.00% |
| Total | 100% | 3.30% | 100.00% | 6.80% |

The attribution results can then be calculated as:

| | |
|---|---|
| Sector Selection | 0.40% |
| Security Selection | 2.80% |
| Interaction | 1.00% |
| Total Excess Return | 4.20% |

Note that the security selection did not change from the previous example. Now consider what happens if the tiny allocation to sector 1 returned 4% instead of 2%. The results are now

| | |
|---|---|
| Sector Selection | 0.40% |
| Security Selection | 3.20% |
| Interaction | 0.60% |
| Total Excess Return | 4.20% |

The security selection has increased by 0.4, whereas the interaction has decreased a compensating amount. If the return of sector 1 had been 50%--the security selection would now be 12.4% and the Interaction would be −8.6%. Is this answer useful?

This problem arises because the Brinson method is calculating the security selection as the excess return that could have been earned had the portfolio sector weights matched the benchmark sector weights. However, this is a hypothetical quantity that can vary tremendously from the excess return that was actually earned in a sector. It seems more logical that the sector selection should take into account the weights allocated to sectors. If a manager has allocated little or nothing to a sector then the return in that sector should be irrelevant. If a manager makes a large allocation to a sector then the return obtained in that sector should be very important.

Another problem we see with the Brinson method is that the definition of sector selection can award positive sector selection for a manager overweighting a sector that underperformed the total benchmark (this does not apply if the Brinson variation described in note 3 is used). For example, suppose a sector had a benchmark return of 1% but the overall benchmark returned 5%. Is overweighting this sector contributing to portfolio excess return? According to the Brinson method it is, simply because the sector has positive return.

## The Parilux Method

Parilux recommends a method that to the author's knowledge was developed by Dr. Ganlin Xu working at Daiwa Securities in about 1991 under the supervision of Dr. Harry Markowitz. This method turns out to be identical to the method recommended by Campisi in 2004. Although this method is similar to the Brinson method, and is based on exactly the same input data, it neatly resolves the problems with the Brinson method that we have discussed in the preceding section. We can develop this method by rearranging equation (2) above to get[6]:

$$\begin{aligned} ER = &\sum_i (w_{pi} - w_{bi})(r_{bi} - r_b) && \text{(Sector Selection)} \\ &+ \sum_i (r_{pi} - r_{bi}) w_{pi} && \text{(Security Selection)} \end{aligned} \tag{5}$$

The security selection for sector $i$ is the outperformance of the benchmark $(r_{pi} - r_{bi})$ times the weight actually allocated to the sector ($w_{pi}$, rather than $w_{bi}$). Defined this way the security selection represents the actual contribution to portfolio excess return that results from outperforming the benchmark sector returns. In other words, if $r_{pi}$ is increased the change in total portfolio excess return will accrue entirely to the security selection for the sector in question. Also, unlike the Brinson method, if the portfolio allocation to a sector is very small, then the sector selection will necessarily be small as well.

The sector selection in sector $i$ is the excess allocation to the sector $(w_{pi} - w_{bi})$ times the benchmark excess return of the sector compared to the total benchmark $(r_{bi} - r_b)$. Now overweighing a sector will result in positive sector selection only if the sector outperformed the broad benchmark.

Probably the best part is there is no need for an interaction term! All of the excess return is attributed to either sector selection or security selection. We believe that in eliminating this interaction term the definitions of security selection and sector selection have become more intuitive and logical as well.

Using the Parilux method, the results of the three examples presented above are

Original example:

[6] In deriving equation (4) it is helpful to note that $\sum_i (w_{pi} - w_{bi}) r_b$ equals zero in all cases since $r_b$ is constant (independent of $i$) and the portfolio and benchmark sector weights must sum to one. Thus subtracting $r_b$ in equation (4) makes no difference to the total sector selection, but results in a more useful allocation of total sector selection to the individual sectors.

| | Sector 1 | Sector 2 | Sector 3 | Portfolio |
|---|---|---|---|---|
| Sector Selection | 0.13% | 0.00% | 0.07% | 0.20% |
| Security Selection | 0.00% | 0.30% | 3.00% | 3.30% |
| Total Excess Return | 0.13% | 0.30% | 3.07% | 3.50% |

Modified example with 0.01% allocated to sector 1 and 69.99% to sector 3:

| | Sector 1 | Sector 2 | Sector 3 | Portfolio |
|---|---|---|---|---|
| Sector Selection | 0.26% | 0.00% | 0.14% | 0.40% |
| Security Selection | 0.00% | 0.30% | 3.50% | 3.80% |
| Total Excess Return | 0.26% | 0.30% | 3.64% | 4.20% |

Sector selection in sector 1 increased because of the reduced the allocation to this underperforming sector. Sector selection in sector 3 increased because of the increased the allocation to the outperforming sector. Also, security selection in sector 3 increased because of the increased allocation to a sector that where the manager exhibited superior security selection.

These results for the last example hold whether sector 1 returns 2%, 4%, or 50% (to within the number of decimal places shown), because the small allocation the sector makes the portfolio return in the sector practically irrelevant.

Note that the total sector selection with the Parilux method is always the same as for the Brinson method, and the total security selection with the Parilux method is the sum of the total Brinson security selection and total interaction terms combined. If we regroup the terms in equation (3) to combine security selection and Interaction we can get

$$\begin{aligned} ER = & \left[\left(\sum_i w_{pi} r_{bi}\right) - r_b\right] && \text{(Sector Selection)} \\ & + \left[r_p - \left(\sum_i w_{pi} r_{bi}\right)\right] && \text{(Security selection)} \end{aligned} \tag{6}$$

While it may not be obvious that the definitions of total security selection and total sector selection in equations 5 and 6 are equivalent, a little algebra does show that they are the same. We feel the presentation in equation 5 is more understandable and also clarifies the contribution of the individual sectors. Please keep in mind that these relations between the total allocations for the Brinson and Parilux methods do not hold for the individual sector allocations.